AC Electric Circuits: The RLC Series Circuit

<u>Goals and Introduction</u>

An AC circuit is an electric circuit in which the current is alternating direction as a function of time. Typically this is accomplished in a continuous fashion where the current increases gradually, reaching its maximum value in the positive direction of flow, and then decreasing, past zero current, until reaching its maximum value in the negative direction. The current will then increase until reaching zero and repeat the entire process again. We model the current as a function of time using a sine function, where *f* is the frequency of the oscillation of the current (Eq. 1).

$$I = I_{\max} \sin\left(2\pi ft\right) \qquad \text{(Eq. 1)}$$

A current in an AC circuit is driven by a power source that provides a potential difference, just as an electric current in a DC circuit. The AC power source, however, must provide a potential difference that oscillates in order to produce a current that oscillates. The frequency of that oscillation, *f*, should be the same as the frequency of the current's oscillation, given that it is the potential difference that causes there to be a current flow in the circuit. However, a strange aspect of AC circuits is that it is not necessarily the case that the potential difference from the source and the current are in phase with each other. That is to say, it is not necessarily the case that when the potential difference of the source is a positive maximum the current is also at a positive maximum value. There can be a *phase difference*, ϕ, or lag/lead, causing the source potential difference and current to reach their respective positive maximum values at different times. Once an AC circuit is connected and operating, this lag/lead will be established and maintained as time goes on. Mathematically, we model the potential difference from the source as a function of time, including the possibility of a phase difference, using Eq. 2.

$$\Delta V_{source} = \Delta V_{source,\max} \sin\left(2\pi ft + \phi\right) \qquad \text{(Eq. 2)}$$

While the current in an AC circuit and the potential difference from the source may not be in sync with each other, it is still true that the maximum potential difference is related to the maximum current in the circuit. In a DC electric circuit, it was the equivalent resistance in the circuit that determined the current through the power source. In an AC circuit, it is a quantity called the *impedance*, *Z*, that relates the maximum potential difference to the maximum current through the source, as seen in Ohm's Law for an AC circuit (Eq. 3). Impedance is measured in units of ohms, like resistance, and is a measure of the net resistive effects of the various elements in an AC circuit.

$$\Delta V_{source,\max} = I_{\max} Z \qquad \text{(Eq. 3)}$$

These strange, but necessary, quantities (phase difference and impedance) for modeling the behavior of an AC circuit arise from the behavior of two particular circuit elements – the capacitor and the inductor.

You may have encountered a capacitor in the lab activity, "DC Electric Circuits – The RC Circuit." A capacitor is constructed from separate conducting surfaces that are able to store equal and opposite amounts of electric charge. In a DC electric circuit with a capacitor and resistor connected to a power source, the capacitor would charge, eventually causing the current in the circuit to become zero, as the potential difference across the capacitor would eventually equal the potential difference of the source. But, in an AC circuit, where the potential difference of the source is oscillating in time, the electric charge on the plates and the potential difference across the capacitor must also oscillate with time. The competing potential differences from the power source and that building up on the capacitor cause an effect where the maximum potential difference across the capacitor and the source lag the behavior of the current. Thus, if only a capacitor and resistor were connected to an AC source, we would expect the maximum value of the current to occur before the maximum value of the potential difference across the source to occur.

An inductor is constructed from a coil of wire that will store energy in a magnetic field created within the inducting coil, because of the current running through it. In a DC electric circuit with an inductor and resistor connected to a power source, the inductor would initially create a back current, due to Faraday's Law, that would cause the circuit to take more time to reach the maximum current than expected. Eventually, there is no back current created by the inductor, once the current in the inductor reaches its maximum value, and becomes constant. In an AC circuit, where the potential difference of the source is oscillating in time, the current is almost always changing, resulting in a back current almost always being created in the circuit, due to the inductor. This would result in a scenario where the potential difference across the inductor and the source lead the behavior of the current. Thus, if only an inductor and resistor were connected to an AC source, we would expect the maximum value of the current to occur after the maximum value of the potential difference across the source to occur.

Given the behaviors described for the capacitor and the inductor in an AC circuit, we can imagine that we might think of each circuit element as having some kind of resistive effect on the circuit. These resistive effects, or *reactances*, depend on the frequency of oscillation of the power source, and thus the current through the circuit elements. The resistive effect of the capacitor in an AC circuit is called the *capacitive reactance,* and the resistive effect of the inductor in an AC circuit is called the *inductive reactance*. These two reactances are given by Eq. 4 and 5 below, where both are measured in the SI unit of ohms, Ω .

$$\chi_C = \frac{1}{2\pi fC} \qquad \text{(Eq. 4)}$$

$$\chi_L = 2\pi fL \qquad \text{(Eq. 5)}$$

These reactances, along with any resistance in the circuit, will determine the impedance and phase difference of an AC circuit, discussed earlier. For an AC circuit with a resistor in series with a capacitor and inductor, the impedance is given by Eq. 6, and the phase difference between the maximum potential difference and the current is given by Eq. 7.

$$Z = \sqrt{R^2 + \ \chi_L - \chi_C \ ^2} \qquad \text{(Eq. 6)}$$

$$\phi = \tan^{-1}\left(\frac{\chi_L - \chi_C}{R}\right) \qquad \text{(Eq. 7)}$$

The derivations of these relationships are beyond the scope of the laboratory introduction, though we should not that the results follow from an application of Kirchoff's Loop Rule to the *RLC* series circuit. It must still be true that the sum of the potential differences across each of the circuit elements at any moment must equal the potential difference from the source at that moment. Needless to say, it would be very difficult to make these kinds of measurements because the potential difference from the source is oscillating so quickly. Rather than measure instantaneous values of the potential difference across any of the circuit elements, we measure what is called the *root mean square* (rms) potential difference across each of the elements. This quantity can be derived from the idea that power is always dissipated in the resistor and stored in the other circuit elements, regardless of the direction of the current flow. Again, this derivation is beyond the scope of this introduction, but we state here that the root mean square of a quantity is related to the maximum value of the quantity by $\sqrt{2}$, as seen in Eq. 8 and 9.

$$\Delta V_{source,\max} = \Delta V_{source,\mathrm{rms}}\sqrt{2} \qquad \text{(Eq. 6)}$$

$$I_{\max} = I_{\mathrm{rms}}\sqrt{2} \qquad \text{(Eq. 7)}$$

Note that we could construct similar equations for the maximum potential difference across any of the circuit elements and the rms potential difference across those elements! Also, given that the same current flows through each circuit element in a series circuit, it must be the case that the rms potential difference across each circuit element is related to the rms value of the current, applying Ohm's law to each case (Eq.'s 8 – 10).

$$\Delta V_{R,\mathrm{rms}} = I_{\mathrm{rms}} R \qquad \text{(Eq. 8)}$$

$$\Delta V_{C,\mathrm{rms}} = I_{\mathrm{rms}} \chi_C \qquad \text{(Eq. 9)}$$

$$\Delta V_{L,\mathrm{rms}} = I_{\mathrm{rms}} \chi_L \qquad \text{(Eq. 10)}$$

In today's lab, you will build an *RLC* series circuit and connect it to an electric signal generator that will function as an AC power source. During the experiment, you will measure the rms potential differences across the three different circuit elements and the power source. With these values, and the known frequency of the source, you can verify that the same current exists in each circuit element by determining the rms current in each element. You can also predict the impedance using the rms potential difference of the source with the mean value of rms current found in each circuit element, and compare it to the predicted impedance.

Goals: (1) Measure and consider the properties of circuit elements in an AC circuit

(2) Test to see the current is the same through each circuit element in an AC series circuit

(3) Predict the impedance of a circuit and perform measurements to test the result.

Procedure

Equipment – electric connection board, 4 wires, AC voltage function generator with wire leads, a resistor, a capacitor, an inductor, two alligator clips, digital multimeter

1) Plug one wire into the "COM" port of the multimeter and another into the "V- - -" port. Attach the alligator clips to the free ends of the wires plugged into the multimeter, and clip them across the two ends of the resistor. Turn the knob on the multimeter to the area marked with Ω and set the dial at "2K." Note that when you are on a setting with a "K", the meter is reading in thousands of Ohms. If the meter shows a one with a line next to it, this means the resistance is larger than the current setting. If this is the case, turn the dial one click counter-clockwise until you can get a reading of the resistance. **Record** the resistance of the resistor, being sure to convert your measurement to ohms.

2) **Record** the capacitance of the capacitor and the inductance of the inductor. There is no meter for this measurement. The values should be printed on the circuit elements themselves. It should be the case that the capacitance is $0.1\ \mu\mathrm{F} = 0.1\times10^{-6}\ \mathrm{F}$, and the inductance should be $10\ \mathrm{mH} = 10\times10^{-3}\ \mathrm{H}$, but check. If they are different, record those values and use them instead of those printed here for this experiment (again, this should not be the case but ask your TA if you are uncertain).

3) Plug in the function generator and turn it on. You should select the button that looks like it has a picture of a nice sine wave underneath it. You should also select the frequency button that says “10k” which means that the meter reading will be in kilohertz (kHz). Be sure that when you record frequencies that you convert the values to hertz, Hz. Turn the knob labeled “AMPL,” or “amplitude,” all the way counterclockwise. Connect the coaxial connector to the output from the meter.

4) Construct the circuit as seen below in Figure 1. You should pine down the resistor in two posts, the inductor in two posts, and the capacitor in two posts. Then, run one wire from one end of the resistor to one end of the inductor, and another wire from the other end of the inductor to one end of the resistor. Then, connect the two leads from the coaxial cable from the power supply to the free end of the resistor and the free end of the capacitor. Run the black wire from the coaxial to the capacitor free end and the other colored wire (likely red) to the free end of the resistor. This should be a circuit with the resistor, inductor, and capacitor in series.

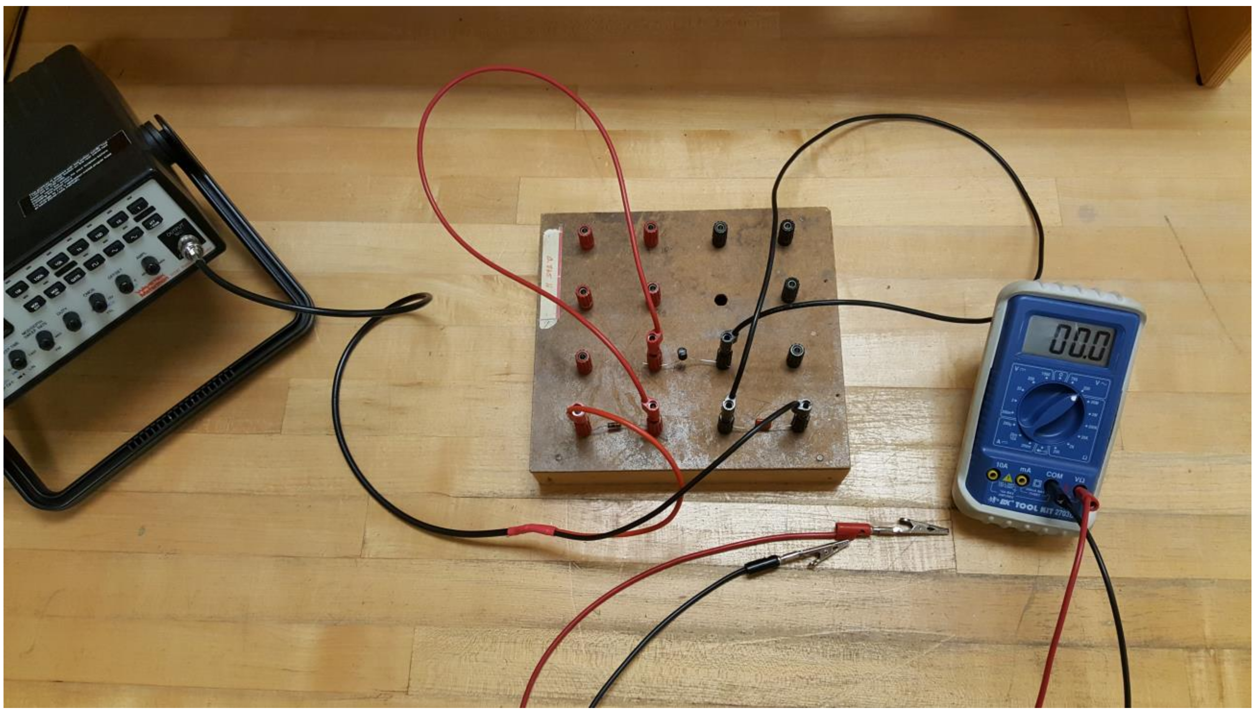

Figure 1

5) Turn the dial on the meter so that it points to “200” in the section labeled as “V ~” on the outer edge of the meter. You don’t need to use the alligator clips any longer, though if you would like to clip near the posts on either end of the circuit elements when recording data you may.

6) Adjust the frequency knob on the AC source so that you have a frequency of about 2000 Hz. Note that you will likely not be able to set this to exactly 2000 Hz. Get it as close to this value as you can and **record** the frequency.

7) Connect the wire from the COM port on the multimeter to the capacitor where the power supply is connected and the other multimeter wire to the resistor where the power supply is connected. Turn the "AMPL," or "amplitude," knob on the AC source until the meter reads at least 6 V. Choose an output level and **record** this meter reading. This is the rms potential difference of the source.

8) One by one, connect the leads from the multimeter across each circuit element alone, so that you can **record** the rms potential difference across the resistor, the inductor, and the capacitor.

9) Repeat steps 6-8, using a frequency of about 10,000 Hz.

10) Repeat steps 6-8 two more times using frequencies of your choice.

11) You may have noticed that the rms potential difference across the inductor was greater than that of the capacitor in one case and then the other way around in another. There is a particular frequency where the rms potential difference across the inductor should be equal to that across the capacitor. This is called the resonant frequency of the *RLC* series circuit. It is also tru that the potential difference across the resistor should be a maximum when this is found. See if you can find that frequency! **Record** the process you went through in determining this frequency and **record** the resonant frequency that you measured. Be sure to also **record** the rms potential difference across each circuit element and the source. To get you started, set the rms potential difference of the source first using the "AMPL" knob on the AC source, and don't change that during your examination.

As always, be sure to organize your data records for presentation in your lab report, using tables and labels where appropriate.

<u>Data Analysis</u>

For each set of data, you have a different frequency. Compute the capacitive reactance, inductive reactance, impedance, and phase angle, using Eq. 4, 5, 6, and 7, for each set of data.

Question 1: What happened to the phase angle as you increased the frequency during the experiment? Does this make sense considering how the reactance of the inductor and capacitor depend on frequency? Explain your answer.

Compute the rms current for the source and each circuit element in each set of data using Eq. 3, 8, 9, and 10.

Calculate the mean value of the rms current for each set of data separately.

The resonant frequency should occur when the capacitive and inductive reactances are the same. Using the capacitance and the inductance, compute a predicted value for the resonant frequency (equate Eq. 4 and 5 and find *f*).

Question 2: What will happen to the impedance of the circuit when the circuit is at the resonant frequency? Explain your answer. Does your data confirm this is the case?

Error Analysis

In each set of data, calculate the percent difference between each rms current and the mean value for the rms current in that data set. You would benefit from making use of a spreadsheet

$$\%diff = \frac{|I_1 - I_{mean}|}{I_1 + I_{mean} \;/2} \times 100\%$$

Question 3: Examine your results for the percent difference of each rms current with the mean value in that circuit. Was there a circuit element that was consistently more different than another? What might this mean about the value we have assumed for that circuit element (*R*, *L*, or *C*)? based on the rms currents that were measured, could we say whether the true value (*R*, *L*, or *C*) is greater or less than what we used in our calculations? Explain your conclusions.

Questions 4: Assuming you did find evidence that one of the circuit elements has a value (*R*, *L*, or *C*) different than what we assumed based on the printed value, how would this affect the rms current we calculated for the source? What quantity is directly affected by this issue that could have caused that calculation to be off? Was the rms current found through the source always very different than the mean value, or was it closer to the mean in some cases? Explain how a circuit element value being different than reported could affect this result and why this effect might be worse under certain conditions.

Calculate the percent error between the experimental value and predicted value of the resonant frequency.

$$\%error = \frac{\left| f_{\text{predict}} - f_{\text{exp}} \right|}{f_{\text{predict}}} \times 100\%$$

Question 5: Comment on the difference found between the experimental and predicted resonant frequency and how your answers to the previous two questions might provide perspective on why there are differences here.

Questions and Conclusions

Be sure to address Questions 1 through 5 and describe what has been verified and tested by this experiment. What are the likely sources of error? Where might the physics principles investigated in this lab manifest in everyday life, or in a job setting?

Pre-Lab Questions

Please read through all the instructions for this experiment to acquaint yourself with the experimental setup and procedures, and develop any questions you may want to discuss with your lab partner or TA before you begin. Then answer the following questions and type your answers into the Canvas quiz tool for "AC Electric Circuits – The RLC Series Circuit," and submit it before the start of your lab section on the day this experiment is to be run.

PL-1) Gwen measures the rms potential difference across a resistor in an AC circuit to be 4.5 V. If the resistance is 1.00×10^3 Ω, what is the rms current through the resistor? Express your answer in amps, A.

PL-2) In an AC electric circuit with *R*, *L*, and *C* in series, the rms current through each circuit element and the source should be

A) getting smaller as it passes through each circuit element.

B) the same.

C) getting bigger as it passes through each circuit element.

D) different in each case.

PL-3) Patrick is working with an *RLC* series circuit where $R = 1.00 \times 10^3$ Ω, $L = 0.0200$ H, and $C = 2.00 \times 10^{-7}$ F. What is the impedance of the circuit if the frequency is 2.00×10^3 Hz? Express your answer in units of ohms, Ω.

PL-4) Patrick is working with an *RLC* series circuit where $R = 1.00 \times 10^3\ \Omega$, $L = 0.0200$ H, and $C = 2.00 \times 10^{-7}$ F. What is the phase angle of the circuit if the frequency is 2.00×10^3 Hz? Express your answer in degrees.

PL-5) Gwen measures the rms potential difference across each circuit element and the AC source. She had though that the sum of the individual rms potential differences would equal the rms potential difference of the source, but it does not. Choose the correct explanation of why this is the case below.

A) The rms potential difference must be made negative in order to show that the sum equals the rms potential difference of the source.

B) The rms potential differences of the capacitor and the inductor must be made negative before being added to the rms potential difference for the resistor. Then, the result would match the rms potential difference for the AC source.

C) She must have made a mistake in measuring the rms potential difference in one or more cases.

D) The rms potential differences are not the instantaneous potential differences. It is only the sum of the instantaneous potential differences that would equal the instantaneous potential difference of the source.